**US Army Corps
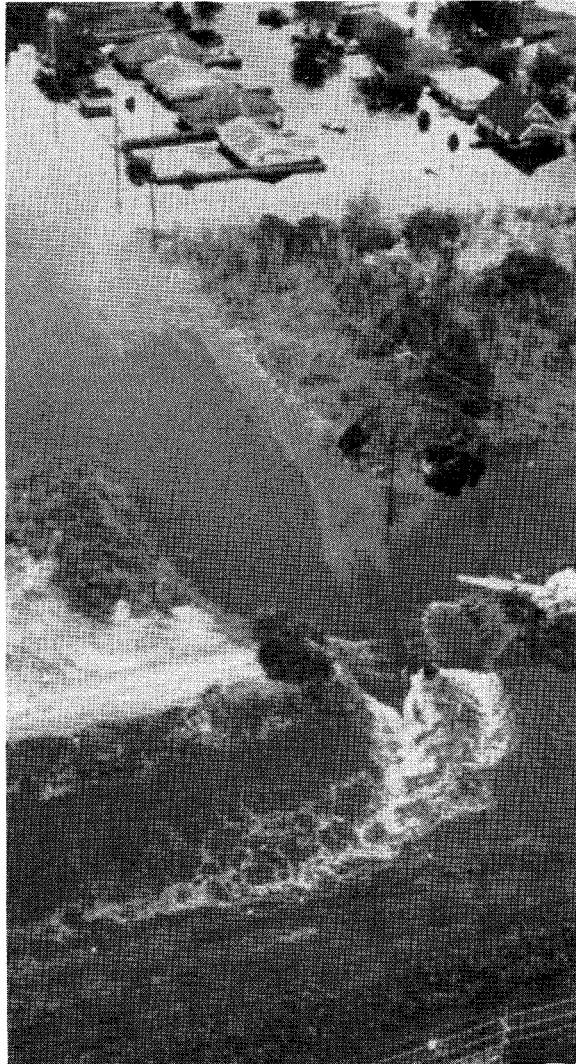of Engineers**

Hydrologic
Risk

Foreword

One of the goals of the U.S. Army Corps of Engineers is to mitigate, in an economically-efficient manner, damage due to floods. Assessment of the risk of flooding is a critical step in deciding how best to accomplish this goal. This pamphlet describes how we in the Corps estimate hydrologic risk, how we use these estimates in project planning, and how the randomness of nature makes our task difficult.

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Summary

Risk is exposure to an undesirable event. Probability is a measure of risk. The probability of a flood is estimated by analyzing historical flood-flow data. The quality of the estimate is related to the quality and quantity of the historical data: a long record of dependable flow data permits us to make a more reliable estimate of the probability of flooding.

Risk estimates play an important role in water-resources project formulation and development by the Corps of Engineers. Our projects are designed to reduce the risk of undesirable economic, environmental, and social consequences of flooding. But floods occur randomly, so the effectiveness of the plans varies from year to year. A project that eliminates all damage and saves many lives one year may not be large enough to eliminate all damage the next year. To resolve this dilemma, we use the long-term average damage as an index of damage potential. This average is computed with estimates of the probability of flooding. Thus the projects are developed considering the consequences of a wide range of floods and the benefits of reducing the adverse impacts.

Introduction

With unlimited funds, the Corps of Engineers could effectively eliminate all existing flood damage at any location in the United States. A giant reservoir or levee could be constructed to control the greatest imaginable flood, eliminating all damage due to that flood and to any smaller flood. Practically, the adverse environmental and social impacts and the cost of such a gigantic project preclude its construction. Fortunately, in most circumstances, a smaller project will reduce significantly the damage at less cost and with minimum environmental impact.

A dilemma exists concerning project size selection: just how much smaller can the project be? Floods occur randomly, so a project designed to eliminate damage due to the 1935 flood may be too small to eliminate all damage in 1988. Likewise the same project may be unused in 1988 if flow is low.

To resolve this dilemma, the Corps considers the risks of flooding, and weighs the consequences of the entire range of possible floods in its development program.

What is risk?

Risk is exposure to an undesirable event. Daily we are confronted with risks. We weigh, with our value systems, the likelihood of the possible outcomes and the benefits and costs of the outcomes, and make decisions accordingly. For example, I know that 48,000 automobile accidents occur annually in the U.S. Nevertheless, I took the risk and drove to work this morning. I felt that the benefits of the action (expediency and comfort) justified taking the risk.

Probability is a numerical index of risk; it is a measure of the likelihood that the undesirable event will occur. If the event is sure to occur, the probability is 1.0, and if it cannot occur, the probability is 0.0.

As a benchmark, the table that follows shows the estimated annual probability of various undesirable events. This table is excerpted from the article by Richard Wilson and E.A.C. Crouch in the 17 April 1987 issue of *Science*.

Event	Annual probability
Electrocution	0.0000053
Airline accident	0.00005
Motor vehicle accident	0.00024
Some form of cancer	0.0028
"100-year" levee overtopping	0.01

No rules exist to explain how we must interpret these probability estimates; events with equal probability aren't equally undesirable, because the impacts aren't equal. For example, I live in an area "protected" by a levee because I enjoy the location. On the other hand, I don't smoke because I feel the risk of cancer is too great. Yet the annual probability of cancer is less than the probability of

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levee overtopping. The decision depends on my perception of the benefits and my aversion to the risk.

How is the probability of flooding estimated?

Probability estimates of undesirable events can be determined by

1. **Subjective weighing.** If you asked a geologist to estimate the probability that you will find rock when you dig in your yard to install a new water line, the estimate provided is likely to be a subjective weighing of the probability, based on experience and intuition.

2. **Analysis of the probabilities of all minor events that contribute to the occurrence of undesirable event.** An example of this is estimation of the probability of a nuclear power plant accident. To make such an estimate, analysts define all the separate events that could lead to an accident, estimate the probability of each, and add the probabilities.

3. **Observation of what has happened in the past.** If the event has occurred in the past, we can use this information to estimate the likelihood that it will occur in the future. This is the approach commonly used to estimate probability of flooding.

Estimating the probability of flooding from historical data is similar to assembling a jigsaw puzzle, without knowledge beforehand of the picture to be produced when assembly is complete. With the puzzle, we draw randomly from the puzzle box a single piece and guess the object shown and the shape of the puzzle. Then we try to assemble the puzzle pieces with this guess in mind. In statistical jargon, the entire set of unassembled pieces is the **parent population,** and each individual piece that we select is a **sample** of the parent population.

In the case of flooding, the parent population is the entire range of flow that could occur. A sample is a

set of flows that have been observed in the past. The goal is to quantify the risk of future flooding based on these observations of historical flooding.

The simplest approach to probability estimation with historical data is to assume that the probability of an event occurring equals the **relative frequency** with which the event occurs historically. The relative frequency of an event can be defined as

$$Relative\ frequency\ =\ \frac{number\ of\ actual\ occurrences}{number\ of\ possible\ occurrences} \quad (1)$$

Suppose, for example, that damage in an area adjacent to a stream begins when flow in the stream exceeds 100,000 cubic feet per second (cfs). To estimate the probability that the annual maximum flow will exceed this value, we could tabulate the maximum value for each year, count the years in which the flow exceeds 100,000 cfs, and divide by the total number of years for which we have observations. For example, if the maximum flow for two years of the last 100 years exceeded 100,000 cfs, we would estimate the annual probability of exceeding 100,000 cfs as 2/100 or 0.02.

The difficulty with estimating flood probability with the relative frequency approach can be illustrated with a simple example: estimating with the relative frequency the probability of rolling a 6 with an honest, 6-sided die. In this case, we know that the true probability is $1/6 = 0.1666$, so the error in our estimates is easy to see. The relative frequency of rolling a 6 with the die is defined as

$$Relative\ frequency\ =\ \frac{number\ of\ times\ 6\ is\ rolled}{number\ of\ times\ the\ die\ is\ rolled} \quad (2)$$

If on the first try, I roll a 6, the relative frequency, according to Eq. 2 is 1/1. We estimate the probability of rolling a 6 as 1. This is not a very

good estimate. Now imagine that on the second try, I roll a 2. The relative frequency now is 1/2, and the probability estimate is 1/2 or 0.5. The estimate still is not very good, but it is getting better.

Fig. 1 shows the results of an experiment in which we rolled a die thousands of times and applied Eq. 1 after each roll to compute the relative frequency estimate of the probability of rolling a 6. In this figure, the relative frequency is plotted as a function of the number of rolls. In these trials, each success (a roll of a 6) has a significant effect on the computed relative frequency. Notice, for example, that in the first 50 or so rolls, a few successive observation of a 6 cause the estimate to increase to 0.23. After 500 rolls, the estimate settles down and approaches 0.16, the true probability of rolling a 6. As the sample size increased, the relative frequency estimate approaches the true probability.

Fig. 1. - Results of Rolling a Die
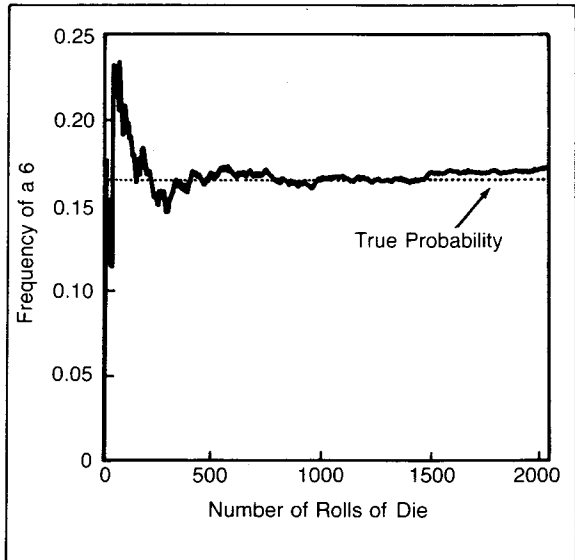
Annual flood probability estimates typically are based on samples of 30 years of data or less. Consequently, relative frequency estimates of probability are greatly influenced by the flow in

one or two years. For example, if we observe annual maximum flow exceeding 100,000 cfs in 2 or 30 years, we may conclude that such flow is common. However, what appears in a small sample to be 2 occurrences in 30 years may, in fact, be 2 occurrences in 300 years if we wait long enough to observe 300 years of flow. Reasonable people won't wait 300 years for data collection for a water-resources project.

An alternative to estimating probability with observed relative frequency is to estimate probability by **statistical inference**. In that case, we study a sample for clues about the behavior of streamflows, and then we describe this behavior with a mathematical function. In statistics, the function is referred to as a **statistical distribution**.

A plot of the statistical distribution of annual maximum streamflow for one stream is shown as Fig. 2.

Fig. 2. - Distribution of Annual Maximum Flow
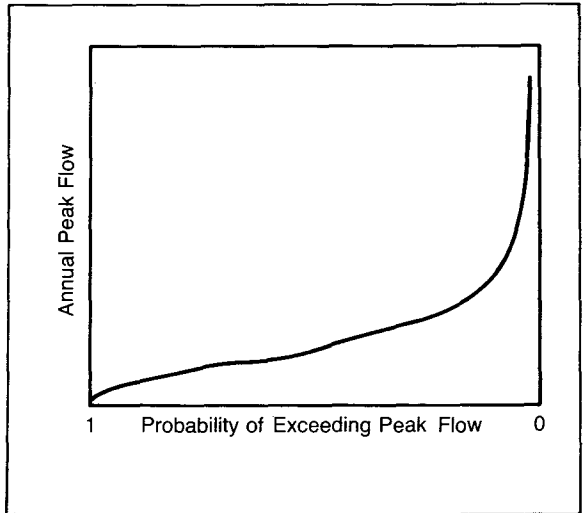
This plot represents a complex equation that relates probability and flow magnitude. With the equation or the plot, we can specify the streamflow magnitude and estimate the probability, or we can

specify the acceptable risk, in terms of probability, and determine the flow for which we should design a flood-control project.

To plot a statistical distribution, we have to describe completely the curve: its shape, location, scale, etc. These characteristics are defined by the **parameters** of the distribution, and they are estimated from a sample. The basic assumption is that the estimates are representative of the entire parent population, so the statistical distribution can be used to estimate probability of future events.

We are faced with two fundamental problems in developing an estimate of the true statistical distribution from an observed sample (Fig. 2).

1. The form of the parent population distribution is never known with certainty, so we don't know if we picked the correct form of the equation from which the plot is drawn.

2. The parameters that represent the parent population must be estimated from a small sample.

To understand better these problems, consider again rolling a die. With a true die with unique faces, we know theoretically that the statistical distribution which defines the probability of rolling any value is

$$Probability = \frac{1}{number\ of\ faces} \qquad (3)$$

In this distribution, the number of faces is a parameter which must be estimated from a sample of the parent population. With 6 faces, the probability is 1/6; with 8 faces, the probability is 1/8, and so on.

Suppose that we couldn't examine the die to count the faces, but that we required an estimate of the

probability of rolling a 6 with the die. We would roll the die, determine the number of unique faces that we have seen, use this as an estimate of the parameter, and solve Eq. 3 to estimate the probability of rolling a 6.

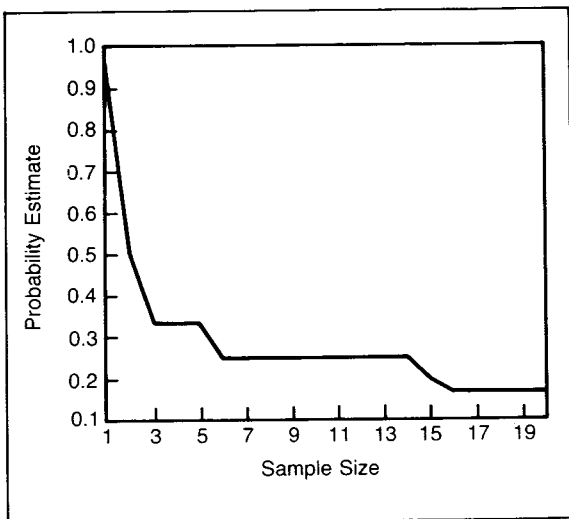**Fig. 3. - Estimates of Probability of Rolling a 6**

Fig. 3 shows the results of an experiment in which we rolled a die, estimated the parameter based on the number of faces seen, and then used this parameter estimate with Eq. 3 to estimate the probability of rolling a 6. The first roll shows one face of the die, so our first estimate is that the die has only one face. This yields an initial estimate that the probability is 1/1 or 1.00. As the die is rolled more, that is, as our sample grows, the estimate of the parameter changes, as does our estimate of the probability.

After 16 rolls of die have been observed, and the probability is estimated as 1/6. More rolls do not turn up more faces, so we feel fairly confident that the estimates are reliable. However, we have no guarantee that a seventh face won't appear after 100 rolls. If we bet the farm that the probability is 1/6, we may regret that decision.

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The statistical distribution commonly used in the U.S. for estimating the probability of a specified annual flood peak is the **log Pearson type III (LP3) distribution**. The distribution has three parameter: a location parameter, a shape parameter, and a scale parameter. Common practice is to estimate these parameters with three indices computed with the sample values: the sample **mean, standard deviation,** and **skew coefficient**. As with the roll of the die, a single sample value can greatly influence the values of the indices, which will, in turn, influence the parameters, which will alter our estimate of the relationship of probability of various flow magnitudes.

To illustrate the impact of a small sample on parameter estimates and the corresponding probability estimates, consider the results of experiments with annual peak flow data for the Dan River. For that river, annual peak flow observations from 1935 to 1982 (48 years) are available. If we estimate the three LP3 distribution parameters with all the available data and use these to compute the annual peak flow value with probability $= 0.01$, we compute 71,700 cubic feet per second (cfs). (The 0.01-probability annual peak flow is often called the 100-year flow.) Now consider what will happen if we have just one less observation; what if the value for 1982 is not available? In that case, the mean, standard deviation, and skew coefficient of the sample are different, so we estimate different values of the location, shape, and scale parameters. The resulting parameter estimates yield an estimated 0.01-probability flow of 69,600 cfs. With this small sample, addition of the observation for 1982 leads to an increase of 2,100 cfs in the estimate of the 0.01-probability annual peak flow.

Fig. 4 shows how the estimated 0.01-probability annual peak flow varies as a function of the sample size for the Dan River. Such decreases or increases with time are not uncommon.

Fig. 4. - Estimates of 0.01- probability Flow, Dan River
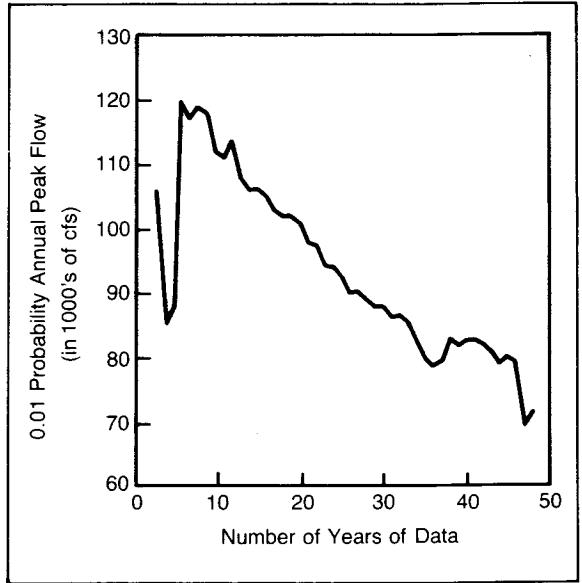
How does the Corps use probability estimates?

The size of a Corps of Engineers flood-control project is selected to maximize the **net economic benefit**. Net economic benefits is computed as

Project benefit
— project cost

net economic benefit   (4)

In the case of flood control, the benefit of a project is the damage reduced, which is computed as

Damage **without** *any action*
— damage **with** *the project*

damage reduced   (5)

In an urban setting, the damage typically is considered a function of the annual maximum flow, so for comparison of alternative size the net benefit is expressed as an annual value. Thus, the best size for the project is the size that maximizes

Annual damage **without** *any action*
— annual damage **with** *the project*
— annual project cost

net annual benefit   (6)

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But, the damage reduced by a project of any size varies randomly from year to year because the flow varies randomly. For example, a levee project designed to eliminate all damage for flows of 100,000 cfs is really larger than required if the flow is only 50,000 cfs. This is a dilemma: how should the annual damage be estimated, given that it changes randomly?

The procedure used by the Corps to estimate annual damage is **expected-value analysis**. This risk-based procedure computes average annual damage considering all possible flow magnitudes, the damage corresponding to each, and the estimated probability of occurrence. The damage is weighed by the probability, and the results are totaled. For example, the damage due to the 0.01-probability flow is weighed by 0.01, and the damage due to the 0.10-probability flow is weighed by 0.10. This weighs the damage due to rare events less than the damage due to frequent events. The results, the **expected annual damage**, is the average yearly damage. With this procedure, the best size for the project is the size that maximizes.

Expected annual damage **without** *any action*
— expected annual damage **with** *the project*
— annual project cost

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net annual benefit (7)

What does this mean to you?

The historical frequency of flooding may not adequately represent the probability of future flooding. Our intuition may deceive us. If we have lived near a stream, we often feel that we have a good knowledge of historical floods. But as the experiments with the die demonstrate, 10, 20, or even 30 years of knowledge may not be sufficient to draw reliable conclusions about probability. A project that is designed so that the expected annual damage-reduction exceeds the cost may seem excessively large in scale, based on our limited exposure. On the other hand, damaging floods in

two successive years may not justify a large expenditure if the annual probability of these floods is low.

The estimates of the flooding probability with a statistical distribution flooding are our best estimates, based on short records. Just as we may draw an incorrect conclusion regarding a jigsaw puzzle picture if we have only a few pieces of the puzzle, so may we draw an incorrect conclusion regarding the statistical distribution of flow. Even if we draw the correct conclusion regarding the distribution, we may estimate incorrectly one or more of the parameters. With a longer record, we have more confidence in our conclusions.

The selected project scale may, in fact, not be the best over the long term. Project scale is a function of flow probability estimates which are used in the expected-value analysis. If these estimates are incorrect, the scale selected may not be the best, and the net benefit of the project may be less than anticipated. The return on investment, in that case, will also be less than expected.

Economic analysis based on statistical analysis is not the sole criterion for plan formulation. The Corps of Engineers historically has recognized the limits on statistical analysis, and has incorporated additional considerations in its water-resources development program. For example, if the risk of loss of life is significant, a levee may be designed to provide a higher level of protection than justified economically.